## Section 8.3

## The Law of Cosines

Case 3: Two sides and the included angle are known (SAS).
Case 4: Three sides are known (SSS).

## THEOREM

## Law of Cosines

For a triangle with sides $a, b, c$ and opposite angles $A, B, C$, respectively,

$$
\begin{aligned}
c^{2} & =a^{2}+b^{2}-2 a b \cos C \\
b^{2} & =a^{2}+c^{2}-2 a c \cos B \\
a^{2} & =b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

1 Solve SAS Triangles

EXAMPLE Using the Law of Cosines to Solve an SAS Triangle
Solve the triangle: $a=2, \quad b=3, \quad C=60^{\circ}$


$$
\begin{aligned}
& c^{2}=a^{2}+b^{2}-2 a b \cos C \\
& c^{2}=2^{2}+3^{2}-2(2)(3) \cos 60^{\circ} \\
& c^{2}=\sqrt{13-12 \cos 60} \\
& c=2.65
\end{aligned}
$$

$$
\begin{aligned}
& 2^{2}=3^{2}+\left(c^{2}-2(3)(0) \cos A\right. \\
& 4=9+7-15.87 \cos A \\
& 4=16-15.87 \cos A \\
& -12=-15.87 \cos A \\
& A=\cos ^{-1} \frac{-12}{15.87} \quad 180-60-40.9=\angle B \\
& A=40.9^{\circ} \quad 79.1^{\circ}=B
\end{aligned}
$$

2 Solve SSS Triangles

EXAMPLE Using the Law of Cosines to Solve an SSS Triangle
Solve the triangle: $\quad a=4, b=3, c=6$

$$
\begin{array}{ll}
4^{2}=3^{2}+6^{2}-2(3)(6) \cos A \\
16=45-36 \cos A & 3^{2}=4^{2}+6^{2}-2(4)(6) \\
-29=-36 \cos A & 9=52-48 \cos B \\
A=\cos ^{-1} \frac{-29}{-36} & -43=-48 \cos B \\
A=36.34^{\circ} & B=\cos ^{-1} \frac{-43}{-48} \\
180-36.34-26.38=117.28^{\circ}=C
\end{array}
$$

