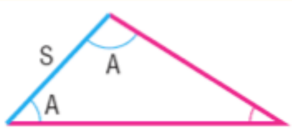
November 28, 2016

Section 8.2

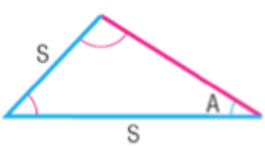
The Law of Sines

CASE 1: One side and two angles are known (ASA or SAA).
CASE 2: Two sides and the angle opposite one of them are known (SSA).
CASE 3: Two sides and the included angle are known (SAS).
CASE 4: Three sides are known (SSS).

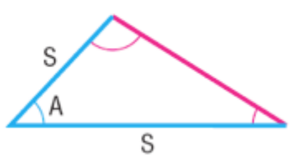


Case 1: ASA

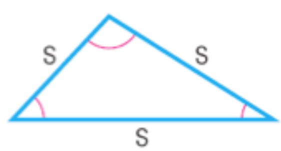
Case 1: SAA



Case 2: SSA



Case 3: SAS



Case 4: SSS

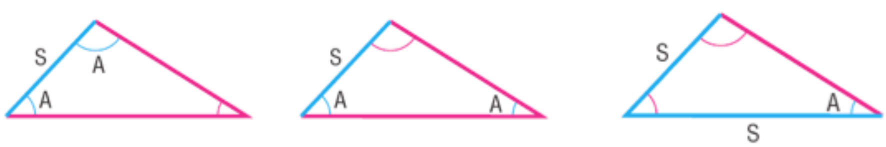
The **Law of Sines** is used to solve triangles for which Case 1 or 2 holds.

THEOREM

Law of Sines

For a triangle with sides a, b, c and opposite angles A, B, C, respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \textbf{(1)}$$

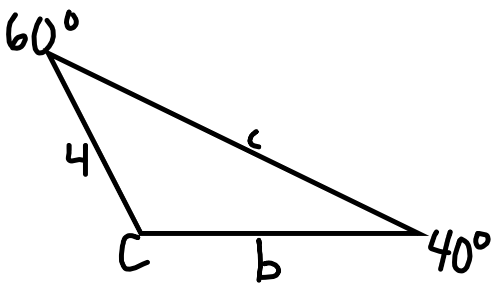


Case 1: ASA Case 1: SAA Case 2: SSA

1 Solve SAA or ASA Triangles

EXAMPLE Using the Law of Sines to Solve an SAA Triangle

Solve the triangle: $A = 40°, B = 60°, a = 4$



$C = 80°$

$$\frac{\sin 40°}{4} = \frac{\sin 80°}{c}$$

$$c = \frac{4 \sin 80°}{\sin 40°}$$

$c = 6.13$

$$\frac{\sin 40°}{4} = \frac{\sin 60°}{b}$$

$$b \sin 40° = 4 \sin 60°$$

$$b = \frac{4 \sin 60°}{\sin 40°}$$

$b = 5.39$

EXAMPLE **Using the Law of Sines to Solve an ASA Triangle**

Solve the triangle: $A = 35°, B = 15°, c = 5$

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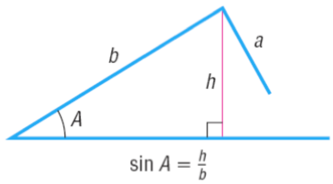
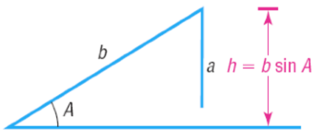
2 Solve SSA Triangles

SSA --- The Ambiguous Case



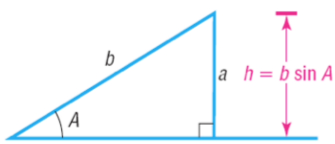
No Triangle If $a < h = b \sin A$, then side a is not sufficiently long to form a triangle. See Figure 14.

Figure 14
$a < h = b \sin A$



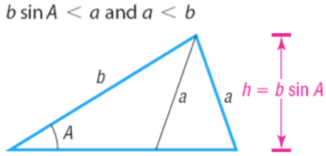
One Right Triangle If $a = h = b \sin A$, then side a is just long enough to form a right triangle. See Figure 15.

Figure 15
$a = h = b \sin A$



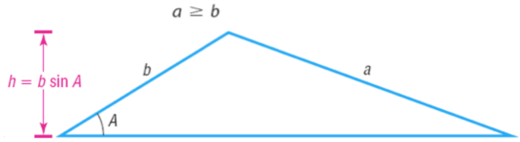
Two Triangles If $h = b \sin A < a$, and $a < b$ two distinct triangles can be formed from the given information. See Figure 16.

Figure 16
$b \sin A < a$ and $a < b$



One Triangle If $a \geq b$, only one triangle can be formed. See Figure 17.

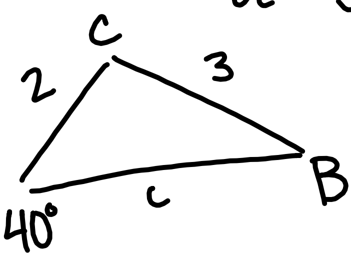
Figure 17
$a \geq b$



EXAMPLE

Using the Law of Sines to Solve an SSA Triangle (One Solution)

$a = 3 \quad b = 2 \quad A = 40^\circ$



$$\frac{\sin 40^\circ}{3} = \frac{\sin B}{2}$$

$$\sin B = \frac{2 \sin 40^\circ}{3}$$

$$B_1 = 25.4^\circ$$

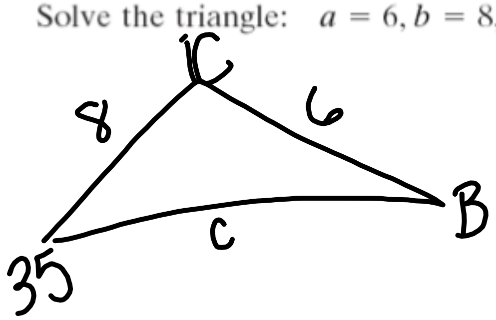
$$B_2 = 180 - 25.4 = 154.6^\circ$$

finish solving Case 1

X Case 2

EXAMPLE **Using the Law of Sines to Solve an SSA Triangle (Two Solutions)**

Solve the triangle: $a = 6, b = 8, A = 35°$



$$\frac{\sin 35°}{6} = \frac{\sin B}{8}$$

$$\sin B = \frac{8 \sin 35°}{6}$$

$$B_1 = 49.9°$$

$$C_1 = 180 - 49.9 - 35 = 95.1°$$

c_1

$$\frac{\sin 35°}{6} = \frac{\sin 95.1}{c}$$

$A = 35°$ $a = 6$ $b = 8$

$$B_2 = 180 - 49.9 = 130.1°$$

$$C_2 = 180 - 35 - 130.1 = 14.9°$$

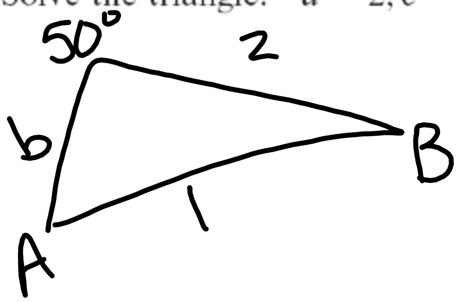
c_2

$$\frac{\sin 35°}{6} = \frac{\sin 14.9}{c}$$

EXAMPLE

Using the Law of Sines to Solve an SSA Triangle (No Solution)

Solve the triangle: $a = 2, c = 1, C = 50°$



$$\frac{\sin 50°}{1} = \frac{\sin A}{2}$$

$$\sin A = 2 \sin 50°$$

$$\sin A = 1.53$$

A = Domain Error

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