Section 7.7

Product-to-Sum and Sum-to-Product Formulas

1 Express Products as Sums

Product-to-Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

EXAMPLE   **Expressing Products as Sums**

Express each of the following products as a sum containing only sines or cosines.

(a) $\sin(3\theta)\sin(7\theta)$ (b) $\cos\theta\cos(5\theta)$ (c) $\sin(2\theta)\cos(7\theta)$

a. $\frac{1}{2}\left[\cos(3\theta - 7\theta) - \cos(3\theta + 7\theta)\right]$

$\frac{1}{2}\left[\cos(-4\theta) - \cos(10\theta)\right]$

$\frac{1}{2}\cos(4\theta) - \frac{1}{2}\cos(10\theta)$

b. $\frac{1}{2}\left[\cos(\theta - 5\theta) + \cos(\theta + 5\theta)\right]$

$\frac{1}{2}\cos(4\theta) + \frac{1}{2}\cos(6\theta)$

c. $\frac{1}{2}\left[\sin(2\theta + 7\theta) + \sin(2\theta - 7\theta)\right]$

$\frac{1}{2}\sin(9\theta) + \frac{1}{2}\sin(-5\theta)$

$\frac{1}{2}\sin(9\theta) - \frac{1}{2}\sin(5\theta)$

2 Express Sums as Products

Sum-to-Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

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EXAMPLE **Expressing Sums (or Differences) as a Product**

Express each sum or difference as a product of sines and/or cosines.

(a) $\sin(4\theta) - \sin(6\theta)$ (b) $\cos(2\theta) + \cos(8\theta)$

a. $2 \sin \frac{4\theta - 6\theta}{2} \cos \frac{4\theta + 6\theta}{2}$

$2 \sin(-\theta) \cos(5\theta)$

$-2 \sin\theta \cos(5\theta)$

b. $2 \cos \frac{2\theta + 8\theta}{2} \cos \frac{2\theta - 8\theta}{2}$

$2 \cos(5\theta) \cos(3\theta)$