

# Section 7.6

## Double-angle and Half-angle Formulas

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \quad \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2} \quad \tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

### THEOREM

#### Half-angle Formulas

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

#### Half-angle Formulas for $\tan \frac{\alpha}{2}$

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

**EXAMPLE** Finding Exact Values Using Half-angle FormulasUse a Half-angle Formula to find the exact value of: (a)  $\sin 22.5^\circ$  (b)  $\cos \frac{5\pi}{12}$ 

$$a. \sin 22.5^\circ = \sin \frac{45^\circ}{2} = \frac{+}{-} \sqrt{\frac{1 - \cos 45^\circ}{2}}$$

 $22.5^\circ \rightarrow Q1$  $\sin + \rightarrow Q1$ 

$$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2 - \sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{2} \cdot \frac{1}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \frac{\sqrt{2 - \sqrt{2}}}{2} \star$$

$$b. \cos \frac{5\pi}{12} = \cos \frac{\frac{5\pi}{6}}{2} = \frac{+}{-} \sqrt{\frac{1 + \cos \frac{5\pi}{6}}{2}}$$

 $\frac{5\pi}{12} \rightarrow Q1$  $\cos + \rightarrow Q1$ 

$$= \sqrt{\frac{1 + \frac{-\sqrt{3}}{2}}{2} \cdot \frac{1}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$= \frac{\sqrt{2 - \sqrt{3}}}{2}$$

**EXAMPLE** Finding Exact Values Using Half-angle Formulas

If  $\cos \alpha = -\frac{1}{5}$ ,  $\frac{\pi}{2} < \alpha < \pi$ , find the exact value of:

$$\begin{aligned} \text{a. } \sin \frac{\alpha}{2} &= \sqrt{\frac{1 - \cos \alpha}{2}} = \sqrt{\frac{1 + \frac{1}{5}}{2}} = \sqrt{\frac{5+1}{10}} \\ &= \sqrt{\frac{6}{10}} \\ &= \sqrt{\frac{3}{5}} \\ &= \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{15}}{5} \end{aligned}$$

$$\begin{aligned} \text{b. } \cos \frac{\alpha}{2} &= \sqrt{\frac{1 + \cos \alpha}{2}} = \sqrt{\frac{1 - \frac{1}{5}}{2}} = \sqrt{\frac{4}{10}} = \sqrt{\frac{2}{5}} \\ &= \frac{\sqrt{2}}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \text{c. } \tan \frac{\alpha}{2} &= \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \frac{\frac{\sqrt{15}}{5}}{\frac{\sqrt{2}}{\sqrt{5}}} = \frac{\sqrt{15}}{5} \cdot \frac{\sqrt{5}}{\sqrt{2}} \\ &= \frac{\sqrt{15}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} \\ &= \frac{\sqrt{150}}{10} \\ &= \frac{5\sqrt{6}}{10} \\ &= \frac{\sqrt{6}}{2} \end{aligned}$$