Section 7.5

Sum and Difference Formulas

Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Sum and Difference Formulas for Sines

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

1 Use Sum and Difference Formulas to Find Exact Values

EXAMPLE

Using the Sum Formula to Find an Exact Value

Find the exact value of $\cos\frac{7\pi}{12}$.

$\frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$

$\frac{3\pi}{12} + \frac{4\pi}{12}$

$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2} - \sqrt{6}}{4}$ $\quad$ $\frac{\sqrt{2}(1-\sqrt{3})}{4}$

EXAMPLE

Using the Difference Formula to Find an Exact Value

Find the exact value of $\cos 15°$.

$\cos(45° - 30°)$

$\cos 45° \cos 30° + \sin 45° \sin 30°$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

EXAMPLE

Using the Sum Formula to Find an Exact Value

Find the exact value of $\sin \frac{19\pi}{12}$.

$\frac{5\pi}{4} + \frac{\pi}{3}$

$\frac{15\pi}{12} + \frac{4\pi}{12}$

$\sin\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)$

$\sin \frac{5\pi}{4} \cos \frac{\pi}{3} + \cos \frac{5\pi}{4} \sin \frac{\pi}{3}$

$-\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$\frac{-\sqrt{2} - \sqrt{6}}{4}$

EXAMPLE

Using the Difference Formula to Find an Exact Value

Find the exact value of $\cos 40° \cos 80° - \sin 40° \sin 80°$.

$\cos(40° + 80°)$

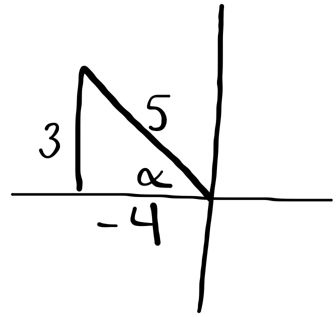
$\cos 120°$

$-\frac{1}{2}$

EXAMPLE **Finding Exact Values**

If it is known that $\sin\alpha = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and that $\sin\beta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$, $\frac{3\pi}{2} < \beta < 2\pi$, find the exact value of

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos(\alpha + \beta)$



$5^2 = 3^2 + x^2$

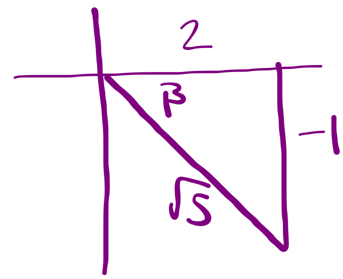
$25 = 9 + x^2$

$16 = x^2$

$4 = x$

a. $\cos\alpha = -\frac{4}{5}$

b. $\cos\beta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$



$(\sqrt{5})^2 = x^2 + (-1)^2$

$5 = x^2 + 1$

$4 = x^2$

$2 = x$

c. $\cos(\alpha + \beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-\frac{4}{5} \cdot \frac{2\sqrt{5}}{5} - \frac{3}{5} \cdot -\frac{\sqrt{5}}{5}$

$\frac{-8\sqrt{5} + 3\sqrt{5}}{25}$

$\frac{-5\sqrt{5}}{25}$

$-\frac{\sqrt{5}}{5}$