Chapter 7 Analytic Trigonometry

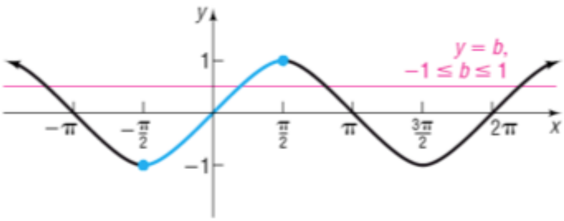
Section 7.1

The Inverse Sine, Cosine, and Tangent Functions

Review of Properties of Functions and Their Inverses

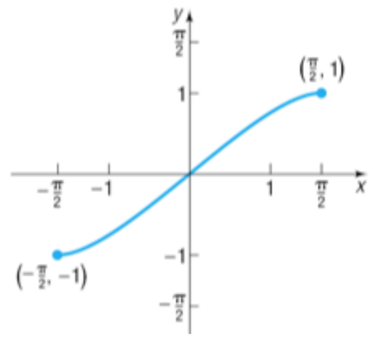
1. $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}.
2. Domain of f = range of f^{-1}, and range of f = domain of f^{-1}.
3. The graph of f and the graph of f^{-1} are symmetric with respect to the line $y = x$.
4. If a function $y = f(x)$ has an inverse function, the equation of the inverse function is $x = f(y)$. The solution of this equation is $y = f^{-1}(x)$.

The Inverse Sine Function



$y = \sin x, -\infty < x < \infty, -1 \le y \le 1$

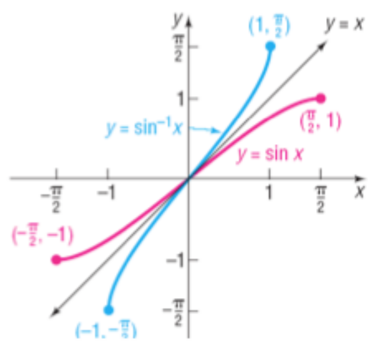
$y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}, -1 \le y \le 1$



$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$
$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$y = \arcsin x$

$y = \sin^{-1} x$



$y = \sin^{-1} x, -1 \le x \le 1, -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

1 Find the Exact Value of an Inverse Sine Function

EXAMPLE **Finding the Exact Value of an Inverse Sine Function**

Find the exact value of: $\sin^{-1} 1 = \frac{\pi}{2}$

$\sin ? = 1$

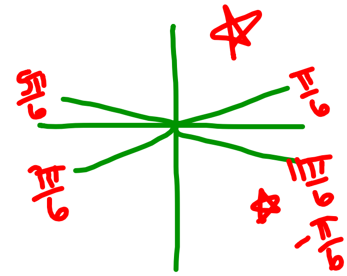
$\sin \frac{\pi}{2} = 1$

EXAMPLE

Finding the Exact Value of an Inverse Sine Function

Find the exact value of: $\sin^{-1}\left(-\frac{1}{2}\right) = \frac{11\pi}{6} = -\frac{\pi}{6}$ $\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin \, ? = -\frac{1}{2}$



D: $-1 \leq x \leq 1$

R: $-\frac{\pi}{2} \leq y < \frac{\pi}{2}$

2 Find an Approximate Value of an Inverse Sine Function

EXAMPLE

Finding an Approximate Value of an Inverse Sine Function

Find an approximate value of:

calculator

(a) $\sin^{-1}\frac{1}{3}$ (b) $\sin^{-1}\left(-\frac{1}{4}\right)$

Express the answer in radians rounded to two decimal places.

a. .34 b. −.25

3 Use Properties of Inverse Functions to Find Exact Values of Certain Composite Functions

Properties of Inverse Functions

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x, \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

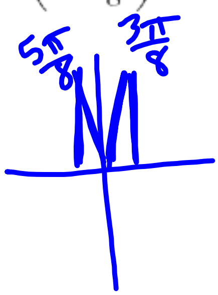
$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x, \qquad \text{where } -1 \le x \le 1$$

EXAMPLE **Finding the Exact Value of Certain Composite Functions**

Find the exact value of each of the following composite functions:

(a) $\sin^{-1}\left(\sin\frac{\pi}{8}\right) = \frac{\pi}{8}$

(b) $\sin^{-1}\left(\sin\frac{5\pi}{8}\right) = \frac{3\pi}{8}$



EXAMPLE

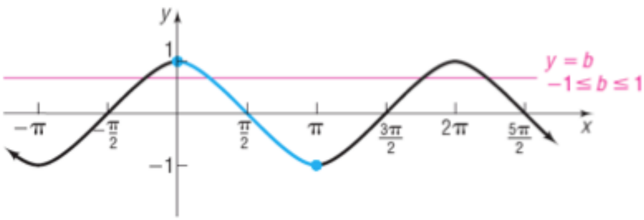
Finding the Exact Value of Certain Composite Functions

Find the exact value, if any, of each composite function.

(a) $\sin(\sin^{-1} 0.5)$ (b) $\sin(\sin^{-1} 1.8)$

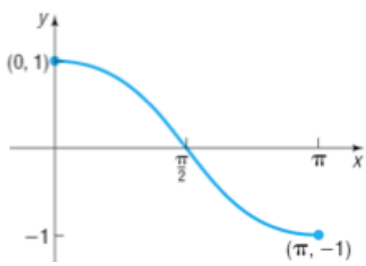
.5 does not exist

The Inverse Cosine Function

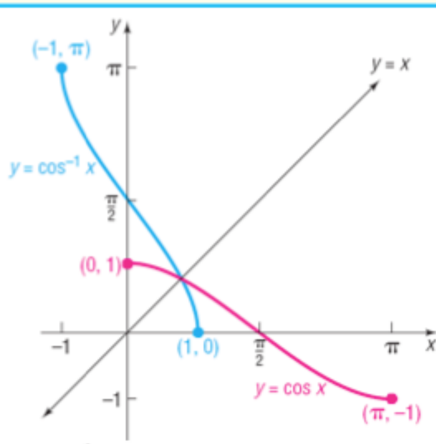


$y = \cos x,\ -\infty < x < \infty,$
$-1 \leq y \leq 1$

$y = \cos x,\ 0 \leq x \leq \pi,\ -1 \leq y \leq 1$



$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y$$
$$\text{where} \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi$$



$y = \cos^{-1} x,\ -1 \leq x \leq 1,\ 0 \leq y \leq \pi$

EXAMPLE

Finding the Exact Value of an Inverse Cosine Function

Find the exact value of: $\cos^{-1} 0 = \frac{\pi}{2}$

$\cos ? = 0$

Find the exact value of: $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

Properties of Inverse Functions

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x, \qquad \text{where } 0 \le x \le \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x, \qquad \text{where } -1 \le x \le 1$$

EXAMPLE **Using Properties of Inverse Functions to Find the Exact Value of Certain Composite Functions**

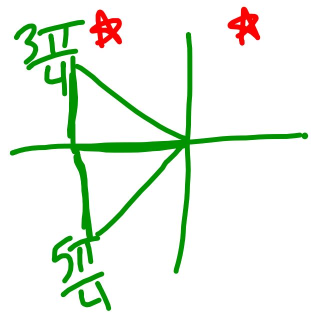
Find the exact value of:

(a) $\cos^{-1}\left[\cos\left(\frac{5\pi}{6}\right)\right] = \frac{5\pi}{6}$

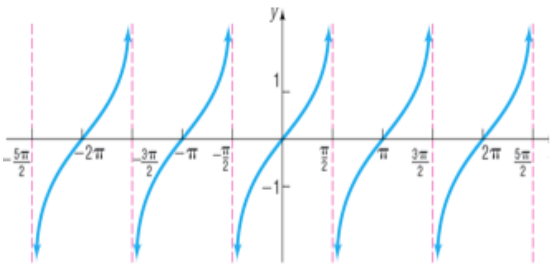
(b) $\cos\left(\cos^{-1} 0.2\right) = .2$

(c) $\cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right] = \frac{3\pi}{4}$

(d) $\cos\left(\cos^{-1} 2\right)$ does not exist

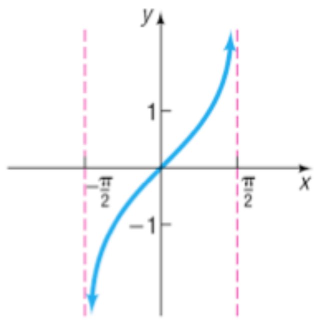


The Inverse Tangent Function

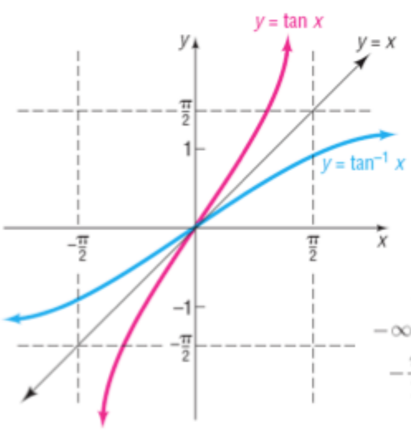


$y = \tan x$, $-\infty < x < \infty$, x not equal to odd multiples of $\frac{\pi}{2}$, $-\infty < y < \infty$

$y = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $-\infty < y < \infty$



$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$
$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$



$y = \tan^{-1} x$, $-\infty < x < \infty$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$

EXAMPLE

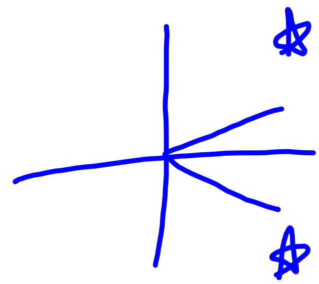
Finding the Exact Value of an Inverse Tangent Function

Find the exact value of: $\tan^{-1} 1 = \frac{\pi}{4}$

$\tan ? = 1$

Find the exact value of: $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

$\tan ? = -\sqrt{3}$



Properties of Inverse Functions

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where } -\infty < x < \infty$$