Section 6.3

Properties of the Trigonometric Functions

4 Find the Values of the Trigonometric Functions Using Fundamental Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

EXAMPLE

Finding Exact Values Using Identities When Sine and Cosine Are Given

Given $\sin\theta = \frac{\sqrt{10}}{10}$ and $\cos\theta = \frac{3\sqrt{10}}{10}$, find the value of each of the four remaining trigonometric functions of θ.

$$\frac{\cancel{\sqrt{10}}}{\cancel{10}} \cdot \frac{\cancel{10}}{3\cancel{\sqrt{10}}} = \frac{1}{3} = \tan\theta$$

$$\frac{10}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \sqrt{10} = \csc\theta$$

$$\frac{10}{3\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{3} = \sec\theta$$

$$3 = \cot\theta$$

The equation of the unit circle is $x^2 + y^2 = 1$

But $y = \sin\theta$ and $x = \cos\theta$, so

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

Fundamental Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

EXAMPLE **Finding the Exact Value of a Trigonometric Expression Using Identities**

Find the exact value of each expression. Do not use a calculator.

(a) $\dfrac{1}{\csc^2 35°} + \cos^2 35°$ (b) $\dfrac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} - \cot\frac{\pi}{3}$

$\sin^2 35° + \cos^2 35° = 1$

$\cot\frac{\pi}{3} - \cot\frac{\pi}{3} = 0$

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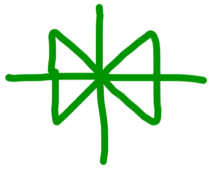
5 Find the Exact Values of the Trigonometric Functions of an Angle Given One of the Functions and the Quadrant of the Angle

EXAMPLE Bowtie



Solution 1
Using a Circle

Finding Exact Values Given One Value and the Sign of Another

Given that $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0$, find the exact value of each of the remaining five trigonometric functions of θ.

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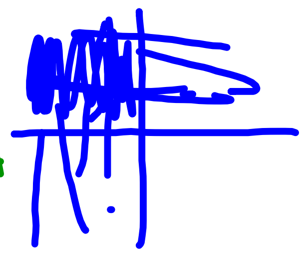

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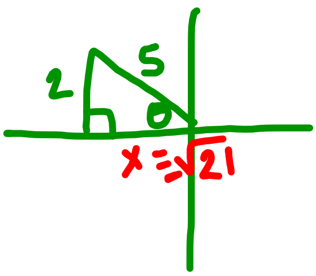


$2^2 + x^2 = 5^2$

$x^2 = 21$

$x = \sqrt{21}$

$\cos\theta = -\frac{\sqrt{21}}{5}$

$\tan\theta = -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$

$\csc\theta = \frac{5}{2}$

$\sec\theta = -\frac{5}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$

$\cot\theta = -\frac{\sqrt{21}}{2}$

Finding the Values of the Trigonometric Functions of θ When the Value of One Function Is Known and the Quadrant of θ Is Known

Given the value of one trigonometric function and the quadrant in which θ lies, the exact value of each of the remaining five trigonometric functions can be found in either of two ways.

Method 1 Using a Circle of Radius *r*

STEP 1: Draw a circle centered at the origin showing the location of the angle θ and the point $P = (x, y)$ that corresponds to θ. The radius of the circle that contains $P = (x, y)$ is $r = \sqrt{x^2 + y^2}$.

STEP 2: Assign a value to two of the three variables x, y, r based on the value of the given trigonometric function and the location of P.

STEP 3: Use the fact that P lies on the circle $x^2 + y^2 = r^2$ to find the value of the missing variable.

STEP 4: Apply the theorem on page 374 to find the values of the remaining trigonometric functions.

Method 2 Using Identities

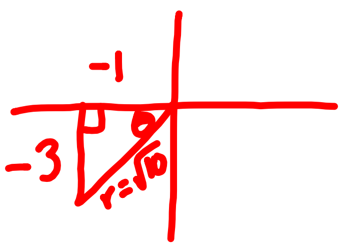
Use appropriately selected identities to find the value of each remaining trigonometric function.

EXAMPLE

Solution 1
Using a Circle

Given the Value of One Trigonometric Function and the Sign of Another, Find the Values of the Remaining Ones

Given that $\cot\theta = \frac{1}{3}$ and $\sin\theta < 0$, find the exact value of each of the remaining five trigonometric functions of θ.



$1^2+3^2=r^2$

$10=r^2$

$\sqrt{10}=r$

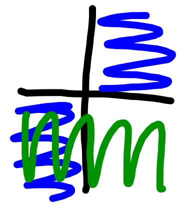
$\sin\theta = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

$\cos\theta = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

$\tan\theta = 3$

$\csc\theta = -\frac{\sqrt{10}}{3}$

$\sec\theta = -\sqrt{10}$



cot +

sin −

QIII