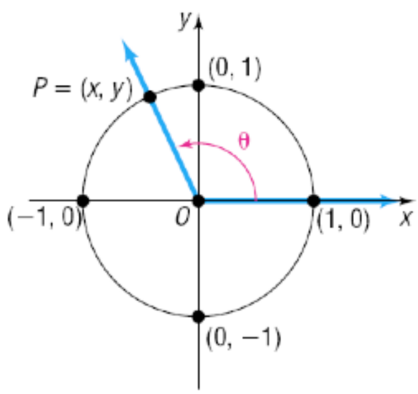
Section 6.3

Properties of the Trigonometric Functions

1 Determine the Domain and the Range of the Trigonometric Functions



$\sin\theta = y$	$\cos\theta = x$	$\tan\theta = \dfrac{y}{x} \quad x \neq 0$
$\csc\theta = \dfrac{1}{y} \quad y \neq 0$	$\sec\theta = \dfrac{1}{x} \quad x \neq 0$	$\cot\theta = \dfrac{x}{y} \quad y \neq 0$

The domain of the sine function is the set of all real numbers.
The domain of the cosine function is the set of all real numbers.

The domain of the tangent function is the set of all real numbers, except odd multiples of $\frac{\pi}{2}$ (90°).

$x \neq \frac{\pi}{2}(2n-1)$
n= integers

The domain of the secant function is the set of all real numbers, except odd multiples of $\frac{\pi}{2}$ (90°).

The domain of the cotangent function is the set of all real numbers, except integer multiples of π (180°).

The domain of the cosecant function is the set of all real numbers, except integer multiples of π (180°).

πn

$$\boxed{-1 \le \sin\theta \le 1 \quad \text{and} \quad -1 \le \cos\theta \le 1}$$

$$\boxed{\csc\theta \le -1 \quad \text{or} \quad \csc\theta \ge 1}$$

Range

$$\boxed{\sec\theta \le -1 \quad \text{or} \quad \sec\theta \ge 1}$$

$$\boxed{-\infty < \tan\theta < \infty \quad \text{and} \quad -\infty < \cot\theta < \infty}$$

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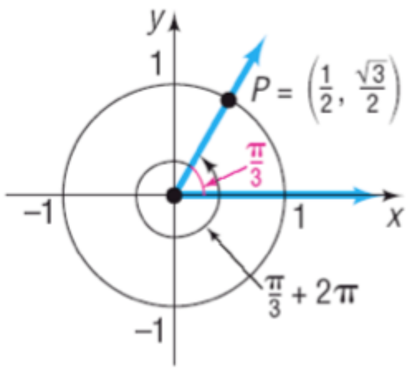
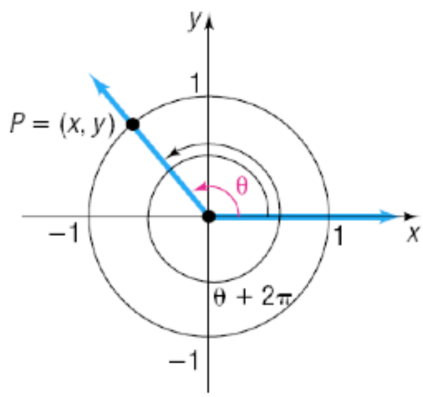
Function	Symbol	Domain	Range
sine	$f(\theta) = \sin\theta$	All real numbers	All real numbers from -1 to 1, inclusive
cosine	$f(\theta) = \cos\theta$	All real numbers	All real numbers from -1 to 1, inclusive
tangent	$f(\theta) = \tan\theta$	All real numbers, except odd integer multiples of $\frac{\pi}{2}(90^\circ)$	All real numbers
cosecant	$f(\theta) = \csc\theta$	All real numbers, except integer multiples of $\pi(180^\circ)$	All real numbers greater than or equal to 1 or less than or equal to -1
secant	$f(\theta) = \sec\theta$	All real numbers, except odd integer multiples of $\frac{\pi}{2}(90^\circ)$	All real numbers greater than or equal to 1 or less than or equal to -1
cotangent	$f(\theta) = \cot\theta$	All real numbers, except integer multiples of $\pi(180^\circ)$	All real numbers

2 Determine the Period of the Trigonometric Functions



When it starts to repeat



$\sin(\theta + 2\pi k) = \sin\theta \qquad \cos(\theta + 2\pi k) = \cos\theta$

where k is any integer

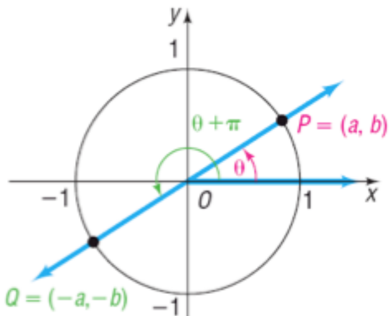
A function f is called **periodic** if there is a positive number p such that, whenever θ is in the domain of f, so is $\theta + p$, and

$$\boxed{f(\theta + p) = f(\theta)}$$

If there is a smallest such number p, this smallest value is called the **(fundamental) period** of f.

Periodic Properties

$\sin(\theta + 2\pi) = \sin\theta$	$\cos(\theta + 2\pi) = \cos\theta$	$\tan(\theta + \pi) = \tan\theta$
$\csc(\theta + 2\pi) = \csc\theta$	$\sec(\theta + 2\pi) = \sec\theta$	$\cot(\theta + \pi) = \cot\theta$



$\tan\theta = \frac{b}{a} = \frac{-b}{-a} = \tan(\theta + \pi)$

EXAMPLE **Finding Exact Values Using Periodic Properties**

Find the exact value of:

(a) $\sin \frac{17\pi}{4}$

$2\pi = \frac{8\pi}{4}$

$\cancel{\frac{8\pi}{4}} + \cancel{\frac{8\pi}{4}} + \frac{\pi}{4} = \frac{17\pi}{4}$

$\sin \frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

(b) $\cos(5\pi)$

$\cancel{2\pi} + \cancel{2\pi} + \pi$

$\cos \pi = \boxed{-1}$

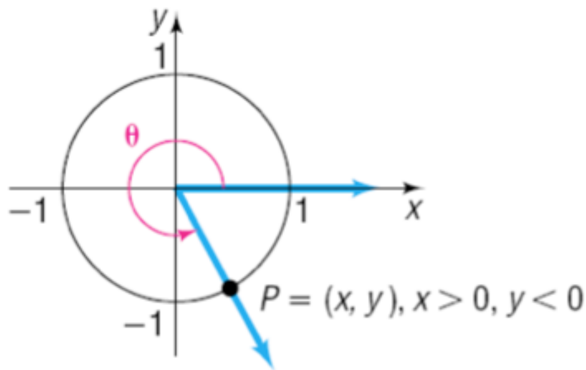
(c) $\tan \frac{5\pi}{4}$

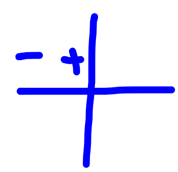
$\pi = \frac{4\pi}{4}$

$\cancel{\frac{4\pi}{4}} + \frac{\pi}{4}$

$\tan \frac{\pi}{4} = \boxed{1}$

3 Determine the Signs of the Trigonometric Functions in a Given Quadrant





$$\sin\theta = y < 0 \qquad \cos\theta = x > 0 \qquad \tan\theta = \frac{y}{x} < 0$$

$$\csc\theta = \frac{1}{y} < 0 \qquad \sec\theta = \frac{1}{x} > 0 \qquad \cot\theta = \frac{x}{y} < 0$$

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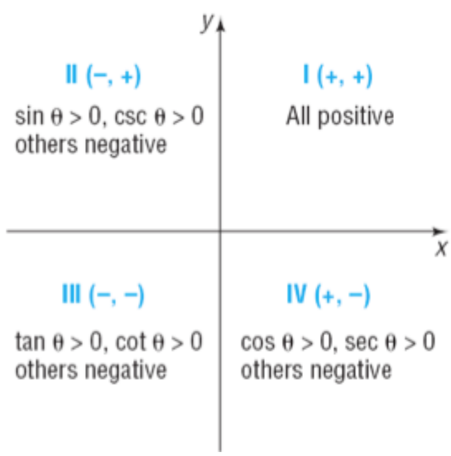
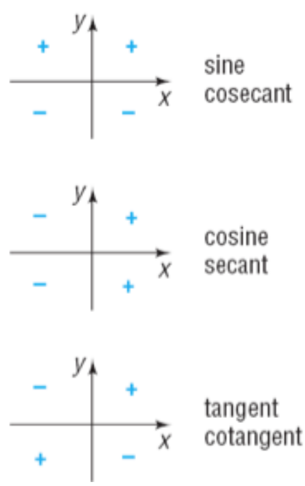
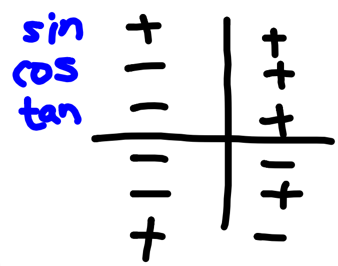
Quadrant of P	$\sin\theta$, $\csc\theta$	$\cos\theta$, $\sec\theta$	$\tan\theta$, $\cot\theta$
I	Positive	Positive	Positive
II	Positive	Negative	Negative
III	Negative	Negative	Positive
IV	Negative	Positive	Negative



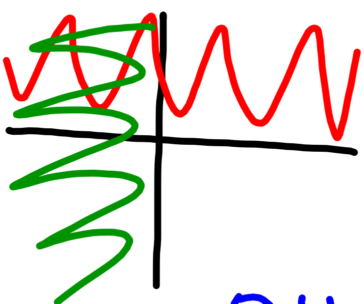




EXAMPLE

Finding the Quadrant in Which an Angle θ Lies

If $\sin\theta > 0$ and $\cos\theta < 0$, name the quadrant in which the angle θ lies.



$\sin > 0$ pos. y

$\cos < 0$ neg. x

QII