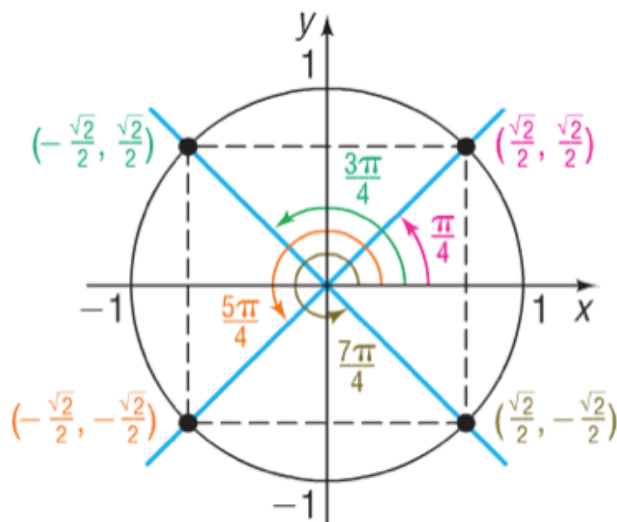


Section 6.2

Trigonometric Functions: Unit Circle Approach

- 5 Find the Exact Values of the Trigonometric Functions
for Integer Multiples of $\frac{\pi}{6} = 30^\circ$, $\frac{\pi}{4} = 45^\circ$, and $\frac{\pi}{3} = 60^\circ$



EXAMPLE

Finding Exact Values for Multiples of $\frac{\pi}{4} = 45^\circ$

Find the exact value of each expression.

- (a) $\cos 135^\circ$ (b) $\tan \frac{3\pi}{4}$ (c) $\sin 225^\circ$ (d) $\cos\left(-\frac{5\pi}{4}\right)$ (e) $\sin \frac{9\pi}{4}$

Q2
x

$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

Q2
x/y

$$\tan \frac{3\pi}{4} = -1$$

Q3
y

$$\sin 225^\circ = -\frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2}$$

Q1	+	+
Q2	-	+
Q3	-	-
Q4	+	-

d. Q2
x

$$\cos\left(-\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

e. Q1
y

$$\sin \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$$

The use of symmetry also provides information about certain integer multiples of the angles $\frac{\pi}{6} = 30^\circ$ and $\frac{\pi}{3} = 60^\circ$. See Figures 29 and 30.

Figure 29

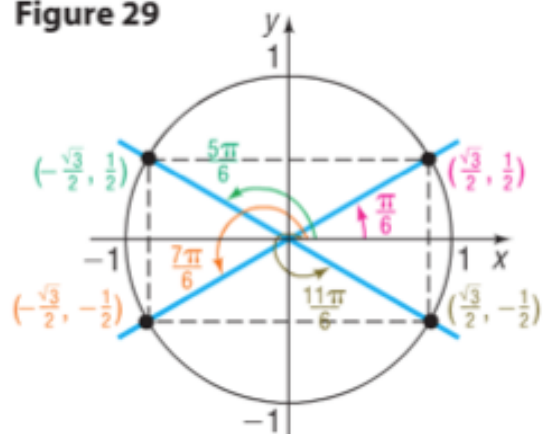
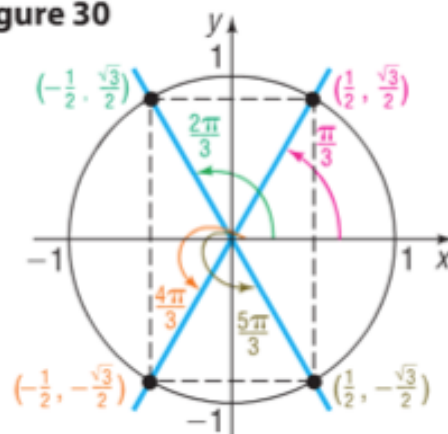


Figure 30



EXAMPLE Finding Exact Values for Multiples of $\frac{\pi}{6} = 30^\circ$ or $\frac{\pi}{3} = 60^\circ$

Find: (a) $\cos 150^\circ$ (b) $\sin(-30^\circ)$ (c) $\tan \frac{4\pi}{3}$ (d) $\sin\left(\frac{7\pi}{6}\right)$

Q2
x
 $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Q4
y
+ -
Q3
x/y
 $\sin(-30^\circ) = -\frac{1}{2} \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Q3
 $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
 $\sin \frac{7\pi}{6} = -\frac{1}{2}$

$\cos 150^\circ = -\frac{\sqrt{3}}{2}$

$\tan \frac{4\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

$\tan \frac{4\pi}{3} = \sqrt{3}$

6 Use a Calculator to Approximate the Value of a Trigonometric Function

Your calculator has buttons for sin, cos, and tan so to find values of the remaining 3 trigonometric functions we use:

$$\sec \theta = \frac{1}{x} = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{y} = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{x}{y} = \frac{1}{\frac{y}{x}} = \frac{1}{\tan \theta}$$

$$\frac{1}{\cos \theta}$$

$$\cos \theta =$$

$$2nd \wedge \boxed{x^{-1}}$$

EXAMPLE**Using a Calculator to Approximate the Value of a Trigonometric Function**

Use a calculator to find the approximate value of:

(a) $\cos 48^\circ$

(b) $\csc 21^\circ$

(c) $\tan \frac{\pi}{12}$

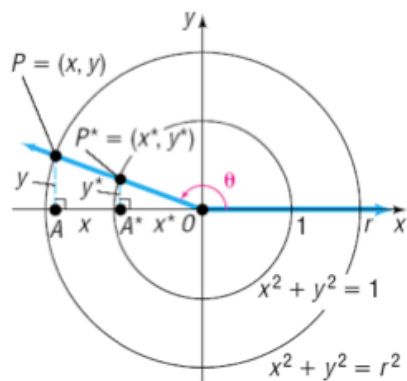
$.6691$

2.7904

$.2679$

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7 Use a Circle of Radius r to Evaluate the Trigonometric Functions



$\cos = \frac{x}{r}$
 $\sin = \frac{y}{r}$

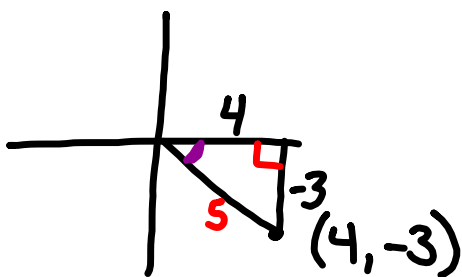
Theorem

For an angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$. Then

$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}, \quad x \neq 0$
$\csc \theta = \frac{r}{y}, \quad y \neq 0$	$\sec \theta = \frac{r}{x}, \quad x \neq 0$	$\cot \theta = \frac{x}{y}, \quad y \neq 0$

Finding the Exact Values of the Six Trigonometric Functions

Find the exact value of each of the six trigonometric functions of a positive angle θ if $(4, -3)$ is a point on its terminal side.



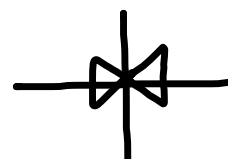
$$r^2 = 4^2 + (-3)^2$$

$$r = 5$$

$$\sin \theta = \frac{-3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = -\frac{3}{4}$$



$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = \frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

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