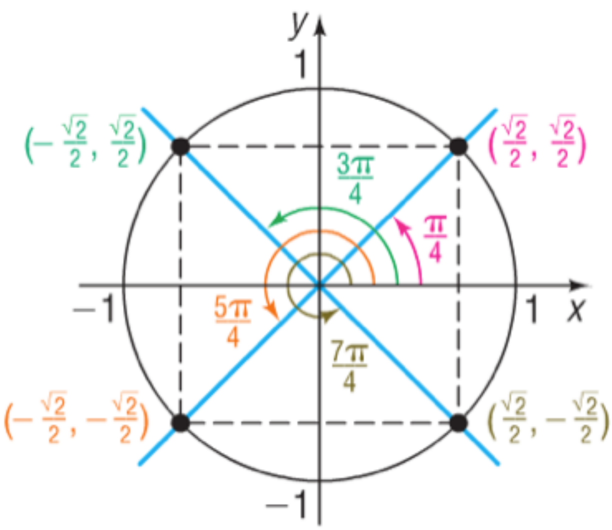
Section 6.2
Trigonometric Functions: Unit Circle Approach

5 Find the Exact Values of the Trigonometric Functions for Integer Multiples of $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, and $\frac{\pi}{3} = 60°$



EXAMPLE **Finding Exact Values for Multiples of $\frac{\pi}{4} = 45°$**

Find the exact value of each expression.

(a) $\cos 135°$ (b) $\tan \frac{3\pi}{4}$ (c) $\sin 225°$ (d) $\cos\left(-\frac{5\pi}{4}\right)$ (e) $\sin \frac{9\pi}{4}$

$\frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}$

Q1 + +
Q2 − +
Q3 − −
Q4 + −

Q2
x

$\cos 135° = -\frac{\sqrt{2}}{2}$

Q2
$\frac{y}{x}$

$\tan \frac{3\pi}{4} = -1$

Q3
y

$\sin 225° = -\frac{\sqrt{2}}{2}$

d. Q2
x

$\cos\left(-\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

e. Q1
y

$\sin \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$

The use of symmetry also provides information about certain integer multiples of the angles $\frac{\pi}{6} = 30°$ and $\frac{\pi}{3} = 60°$. See Figures 29 and 30.

Figure 29

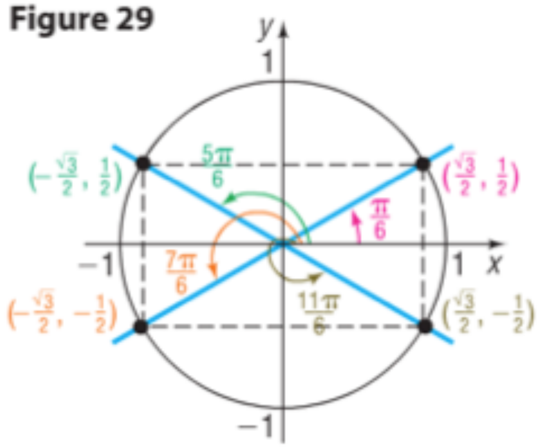
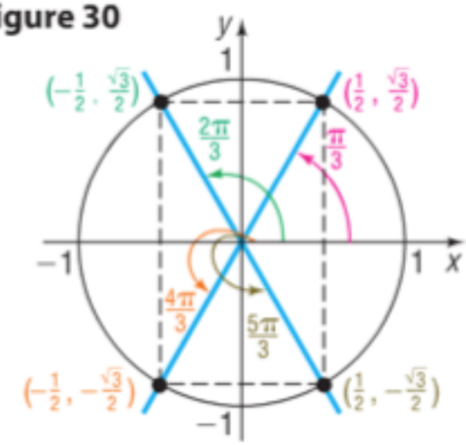


Figure 30



EXAMPLE **Finding Exact Values for Multiples of $\frac{\pi}{6} = 30°$ or $\frac{\pi}{3} = 60°$**

Find: (a) $\cos 150°$ (b) $\sin(-30°)$ (c) $\tan \frac{4\pi}{3}$ (d) $\sin\left(\frac{7\pi}{6}\right)$

Q2

x

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$-$ $+$

$\cos 150° = -\frac{\sqrt{3}}{2}$

Q4

y

$+$ $-$

$\sin(-30)° = -\frac{1}{2}$

Q3

$\frac{y}{x}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$-$ $-$

$\tan \frac{4\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\tan \frac{4\pi}{3} = \sqrt{3}$

Q3

y

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$-$ $-$

$\sin \frac{7\pi}{6} = -\frac{1}{2}$

6 Use a Calculator to Approximate the Value of a Trigonometric Function

Your calculator has buttons for sin, cos, and tan so to find values of the remaining 3 trigonometric functions we use:

$$\sec\theta = \frac{1}{x} = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{y} = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{x}{y} = \frac{1}{\frac{y}{x}} = \frac{1}{\tan\theta}$$

$$\frac{1}{\cos\theta}$$

$\cos\theta =$

2nd ^ $\boxed{x^{-1}}$

EXAMPLE

Using a Calculator to Approximate the Value of a Trigonometric Function

Use a calculator to find the approximate value of:

(a) $\cos 48°$ (b) $\csc 21°$ (c) $\tan \frac{\pi}{12}$

.6691 2.7904 .2679

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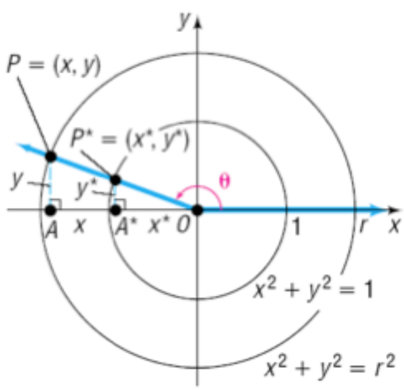
7 Use a Circle of Radius r to Evaluate the Trigonometric Functions



$\cos = \frac{x}{1}$

$\sin = \frac{y}{1}$

Theorem

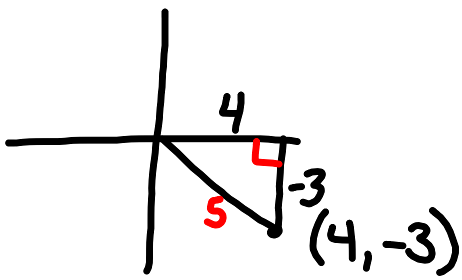
For an angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$. Then

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}, \quad x \neq 0$$

$$\csc\theta = \frac{r}{y}, \quad y \neq 0 \qquad \sec\theta = \frac{r}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

Finding the Exact Values of the Six Trigonometric Functions

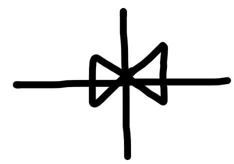
Find the exact value of each of the six trigonometric functions of a positive angle θ if $(4, -3)$ is a point on its terminal side.



$$\sin\theta = \frac{-3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$$\tan\theta = -\frac{3}{4}$$



$$\csc\theta = -\frac{5}{3}$$

$$\sec\theta = \frac{5}{4}$$

$$\cot\theta = -\frac{4}{3}$$

$$r^2 = 4^2 + (-3)^2$$

$$r = 5$$

6.2 Day 3

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47-83 odd

85-105 odd