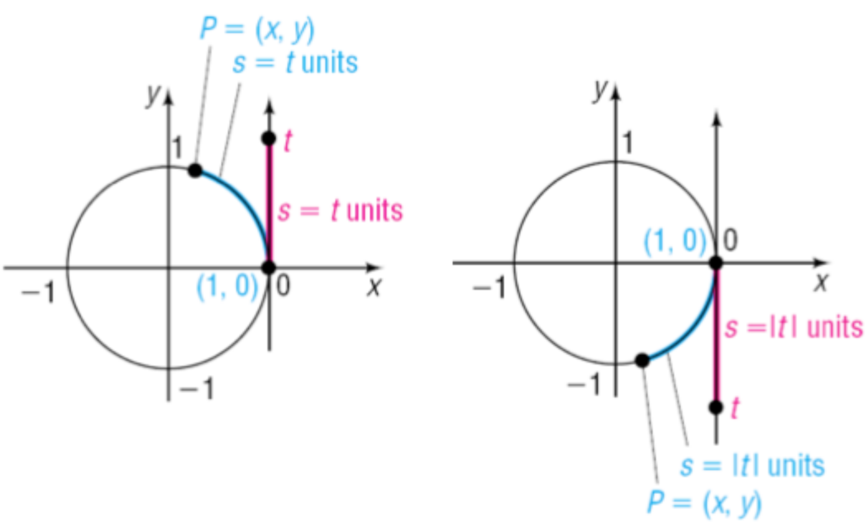
Section 6.2

Trigonometric Functions: Unit Circle Approach

The Unit Circle



Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

The **sine function** associates with t the y-coordinate of P and is denoted by

$$\sin t = y$$

The **cosine function** associates with t the x-coordinate of P and is denoted by

$$\cos t = x$$

(x, y)

cos, sin

If $x \neq 0$, the **tangent function** associates with t the ratio of the y-coordinate to the x-coordinate of P and is denoted by

$$\tan t = \frac{y}{x}$$

Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

If $y \neq 0$, the **cosecant function** is defined as

$$\csc t = \frac{1}{y}$$

sin

If $x \neq 0$, the **secant function** is defined as

$$\sec t = \frac{1}{x}$$

cos

If $y \neq 0$, the **cotangent function** is defined as

$$\cot t = \frac{x}{y}$$

1 Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle

EXAMPLE **Finding the Values of the Trigonometric Functions Using a Point on the Unit Circle**

Find the values of $\sin t$, $\cos t$, $\tan t$, $\csc t$, $\sec t$, and $\cot t$ if $P = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is the point on the unit circle that corresponds to the real number t.

$\sin t = \frac{\sqrt{3}}{2}$

$\cos t = -\frac{1}{2}$

$\tan t = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot -\frac{\cancel{2}}{1} = -\sqrt{3}$

$\csc t = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec t = -2$

$\cot t = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

DEFINITION

If $\theta = t$ radians, the six **trigonometric functions of the angle θ** are defined as

$$\boxed{\begin{array}{lll} \sin\theta = \sin t & \cos\theta = \cos t & \tan\theta = \tan t \\ \csc\theta = \csc t & \sec\theta = \sec t & \cot\theta = \cot t \end{array}}$$

2 Find the Exact Values of the Trigonometric Functions of Quadrantal Angles

EXAMPLE **Finding the Exact Values of the Six Trigonometric Functions of Quadrantal Angles**

Find the exact values of each of the six trigonometric functions of

(a) $\theta = 0 = 0°$ $(1,0)$

$\sin 0 = 0$

$\cos 0 = 1$

$\tan 0 = \frac{0}{1} = 0$

$\csc 0 = \frac{1}{0} =$ undefined

$\sec 0 = 1$

$\cot 0 =$ undefined

(b) $\theta = \frac{\pi}{2} = 90°$ $(0,1)$

$\sin \frac{\pi}{2} = 1$

$\cos \frac{\pi}{2} = 0$

$\tan \frac{\pi}{2} = \frac{1}{0} =$ undefined

$\csc \frac{\pi}{2} = 1$

$\sec \frac{\pi}{2} =$ undefined

$\cot \frac{\pi}{2} = 0$

(c) $\theta = \pi = 180°$ $(-1,0)$

$\sin \pi = 0$

$\cos \pi = -1$

$\tan \pi = \frac{0}{-1} = 0$

$\csc \pi =$ undefined

$\sec \pi = -1$

$\cot \pi =$ undefined

(d) $\theta = \frac{3\pi}{2} = 270°$ $(0,-1)$

$\sin \frac{3\pi}{2} = -1$

$\cos \frac{3\pi}{2} = 0$

$\tan \frac{3\pi}{2} =$ undefined

$\csc \frac{3\pi}{2} = -1$

$\sec \frac{3\pi}{2} =$ undefined

$\cot \frac{3\pi}{2} = 0$

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0°	0	1	0	Not defined	1	Not defined
$\frac{\pi}{2}$	90°	1	0	Not defined	1	Not defined	0
π	180°	0	-1	0	Not defined	-1	Not defined
$\frac{3\pi}{2}$	270°	-1	0	Not defined	-1	Not defined	0

EXAMPLE **Finding Exact Values of the Trigonometric Functions of Angles That Are Integer Multiples of Quadrantal Angles**

Find the exact value of:

(a) $\sin(3\pi)$

$\sin 3\pi = 0$

(b) $\cos(-270°)$

$\cos(-270°) = 0$