

5-6

Inequalities in One Triangle

Content Standard
 Extends G.CO.10 Prove theorems about triangles . . .

Objective To use inequalities involving angles and sides of triangles

Essential Understanding The angles and sides of a triangle have special relationships that involve inequalities.

Take note

Property Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$.

Take note

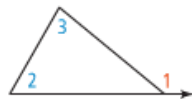
Corollary Corollary to the Triangle Exterior Angle Theorem

Corollary

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.

If . . .

$\angle 1$ is an exterior angle



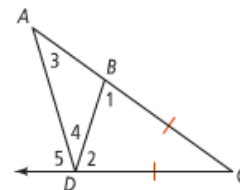
Then . . .

$m\angle 1 > m\angle 2$ and
 $m\angle 1 > m\angle 3$

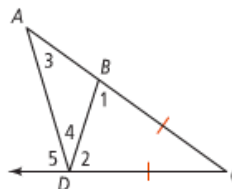


Problem 1 Applying the Corollary

Use the figure at the right. Why is $m\angle 2 > m\angle 3$?



Got It? 1. Why is $m\angle 5 > m\angle C$?



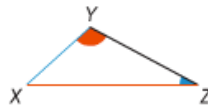
Take note

Theorem 5-10**Theorem**

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

If . . .

$$XZ > XY$$



Then . . .

$$m\angle Y > m\angle Z$$

You will prove Theorem 5-10 in Exercise 40.

**Problem 2 Using Theorem 5-10**

A town park is triangular. A landscape architect wants to place a bench at the corner with the largest angle. Which two streets form the corner with the largest angle?



- Got It?** 2. Suppose the landscape architect wants to place a drinking fountain at the corner with the second largest angle. Which two streets form the corner with the second-largest angle?



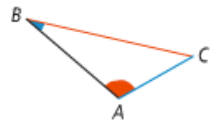
Take note

Theorem 5-11**Theorem**

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

If ...

$$m\angle A > m\angle B$$

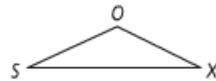


Then ...

$$BC > AC$$



Got It? 3. Reasoning In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Explain your reasoning.



Fake note

Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$XY + YZ > XZ \quad YZ + XZ > XY \quad XZ + XY > YZ$$



You will prove Theorem 5-12 in Exercise 45.



Got It? 4. Can a triangle have sides with the given lengths? Explain.

a. 2 m, 6 m, and 9 m

b. 4 yd, 6 yd, and 9 yd



Got It? 5. A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?