# 2-5 Reasoning in Algebra and Geometry

**Objective** To connect reasoning in algebra and geometry

**Essential Understanding** Algebraic properties of equality are used in geometry. They will help you solve problems and justify each step you take.

In geometry you accept postulates and properties as true. Some of the properties that you accept as true are the properties of equality from algebra.

take note

**Key Concept** Properties of Equality

Let $a$, $b$, and $c$ be any real numbers.

| | |
|---|---|
| **Addition Property** | If $a = b$, then $a + c = b + c$. |
| **Subtraction Property** | If $a = b$, then $a - c = b - c$. |
| **Multiplication Property** | If $a = b$, then $a \cdot c = b \cdot c$. |
| **Division Property** | If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. |
| **Reflexive Property** | $a = a$ |
| **Symmetric Property** | If $a = b$, then $b = a$. |
| **Transitive Property** | If $a = b$ and $b = c$, then $a = c$. |
| **Substitution Property** | If $a = b$, then $b$ can replace $a$ in any expression. |

take note

**Key Concept** The Distributive Property

Use multiplication to distribute $a$ to each term of the sum or difference within the parentheses.

Sum:
$$a(b + c) = a(b + c) = ab + ac$$

Difference:
$$a(b - c) = a(b - c) = ab - ac$$

ⓒ **Problem 1** Justifying Steps When Solving an Equation

**Algebra** What is the value of $x$? Justify each step.

$\angle AOM$ and $\angle MOC$ are supplementary.  ⦠ that form a linear pair are supplementary.

| | |
|---|---|
| $m\angle AOM + m\angle MOC = 180$ | Definition of supplementary ⦠ |
| $(2x + 30) + x = 180$ | Substitution Property |
| $3x + 30 = 180$ | Distributive Property |
| $3x = 150$ | Subtraction Property of Equality |
| $x = 50$ | Division Property of Equality |

Diagram: ray with point $M$, $(2x + 30)°$ and $x°$ at points $A$, $O$, $C$.
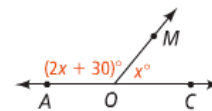
✔ **Got It?** **1.** What is the value of $x$? Justify each step.
**Given:** $\overrightarrow{AB}$ bisects $\angle RAN$.

Diagram: $B$ at top, $x°$ and $(2x - 75)°$ at points $R$, $A$, $N$.

**take note**

## Key Concept   Properties of Congruence

**Reflexive Property**          $\overline{AB} \cong \overline{AB}$     $\angle A \cong \angle A$

**Symmetric Property**       If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

**Transitive Property**        If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.
If $\angle B \cong \angle A$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

ⓒ **Problem 2**   **Using Properties of Equality and Congruence**

What is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

**A** $2x + 9 = 19$
$2x = 10$                              Subtraction Property of Equality

**B** $\angle O \cong \angle W$ and $\angle W \cong \angle L$
$\angle O \cong \angle L$                          Transitive Property of Congruence

**C** $m\angle E = m\angle T$
$m\angle T = m\angle E$                    Symmetric Property of Equality

✅ **Got It?**  **2.** For parts (a)–(c), what is the name of the property of equality or congruence that justifies going from the first statement to the second statement?
**a.** $\overline{AR} \cong \overline{TY}$              **b.** $3(x + 5) = 9$              **c.** $\frac{1}{4}x = 7$
$\overline{TY} \cong \overline{AR}$                    $3x + 15 = 9$                      $x = 28$
**d. Reasoning**  What property justifies the statement $m\angle R = m\angle R$?

A **proof** is a convincing argument that uses deductive reasoning. A proof logically shows why a conjecture is true. A **two-column proof** lists each statement on the left. The justification, or the reason for each statement, is on the right. Each statement must follow logically from the steps before it.

**Problem 3** **Writing a Two-Column Proof**

Write a two-column proof.

**Given:** $m\angle 1 = m\angle 3$

**Prove:** $m\angle AEC = m\angle DEB$
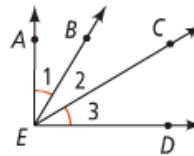
**Know**
$m\angle 1 = m\angle 3$

**Need**
To prove that
$m\angle AEC = m\angle DEB$

**Plan**
Add $m\angle 2$ to both $m\angle 1$ and $m\angle 3$. The resulting angles will have equal measure.

| Statements | Reasons |
|---|---|
| 1) $m\angle 1 = m\angle 3$ | 1) Given |
| 2) $m\angle 2 = m\angle 2$ | 2) Reflexive Property of Equality |
| 3) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$ | 3) Addition Property of Equality |
| 4) $m\angle 1 + m\angle 2 = m\angle AEC$<br>$m\angle 3 + m\angle 2 = m\angle DEB$ | 4) Angle Addition Postulate |
| 5) $m\angle AEC = m\angle DEB$ | 5) Substitution Property |

**Got It?** **3. a.** Write a two-column proof.

**Given:** $\overline{AB} \cong \overline{CD}$

**Prove:** $\overline{AC} \cong \overline{BD}$

**b. Reasoning** In Problem 3, why is Statement 2 necessary in the proof?