

Key Concept Conditional Statements

Definition
A conditional is an if-then statement.
The hypothesis is the part $p$ following if.
The conclusion is the part $q$ following then.

Symbols
$p \rightarrow q$
Read as
"if $p$ then $q$ " or " $p$ implies $q$."

Diagram
$q$
p

## Problem 1 Identifying the Hypothesis and the Conclusion

What are the hypothesis and the conclusion of the conditional?

If an angle measures 130 , then the angle is obtuse.

## Problem 2 Writing a Conditional

How can you write the following statement as a conditional?
Vertical angles share a vertex.
2. How can you write "Dolphins are mammals" as a conditional?

The truth value of a conditional is either true or false.
3. Is the conditional true or false? If it is false, find a counterexample.
a. If a month has 28 days, then it is February.
b. If two angles form a linear pair, then they are supplementary.

The negation of a statement $p$ is the opposite of the statement. The symbol is $\sim p$ and is read "not $p$."

| Statement | How to Write It | Example | Symbols | How to Read It |
| :---: | :---: | :---: | :---: | :---: |
| Conditional | Use the given hypothesis and conclusion. | If $m \angle A=15$, then $\angle A$ is acute. | $p \rightarrow q$ | If $p$, then $q$. |
| Converse | Exchange the hypothesis and the conclusion. | If $\angle A$ is acute, then $m \angle A=15$. | $q \rightarrow p$ | If $q$, then $p$. |
| Inverse | Negate both the hypothesis and the conclusion of the conditional. | If $m \angle A \neq 15$, then $\angle A$ is not acute. | $\sim p \rightarrow \sim q$ | If not $p$, then not $q$. |
| Contrapositive | Negate both the hypothesis and the conclusion of the converse. | If $\angle A$ is not acute, then $m \angle A \neq 15$. | $\sim q \rightarrow \sim p$ | If not $q$, then not $p$. |

Below are the truth values of the related statements above. Equivalent statements have the same truth value.

| Statement | Example | Truth Value |
| :--- | :--- | :--- |
| Conditional | If $m \angle A=15$, then $\angle A$ is acute. | True |
| Converse $\angle A$ is acute, then $m \angle A=15$. | False |  |
| Inverse | If $m \angle A \neq 15$, then $\angle A$ is not acute. | False |
| Contrapositive | If $\angle A$ is not acute, then $m \angle A \neq 15$. | True |

A conditional and its contrapositive are equivalent statements. They are either both true or both false. The converse and inverse of a statement are also equivalent statements.
4. What are the converse, inverse, and contrapositive of the conditional statement below? What are the truth values of each? If a statement is false, give a counterexample.

If a vegetable is a carrot, then it contains beta carotene.

