

5-6

Inequalities in One Triangle

Content Standard
Extends G.CO.10 Prove theorems about triangles . . .

Objective To use inequalities involving angles and sides of triangles

Essential Understanding The angles and sides of a triangle have special relationships that involve inequalities.

Take note

Property Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$.

Take note

Corollary Corollary to the Triangle Exterior Angle Theorem

Corollary

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.

If . . .

$\angle 1$ is an exterior angle



Then . . .

$m\angle 1 > m\angle 2$ and
 $m\angle 1 > m\angle 3$



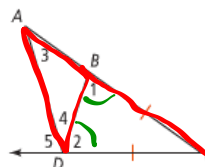
Problem 1 Applying the Corollary

Use the figure at the right. Why is $m\angle 2 > m\angle 3$?

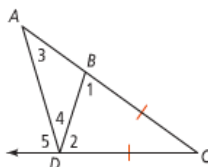
$$m\angle 1 = m\angle 2 \quad m\angle 1 > m\angle 3$$

$$\downarrow$$

$$m\angle 2 > m\angle 3$$



Got It? 1. Why is $m\angle 5 > m\angle C$?



$\angle 5$ is an exterior \angle
 $\angle C$ is a remote interior \angle
 $m\angle 5 > m\angle C$
 Corollary to the Δ Exterior \angle Thm

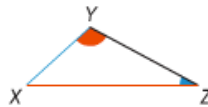
Take note

Theorem 5-10**Theorem**

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

If ...

$$XZ > XY$$



Then ...

$$m\angle Y > m\angle Z$$

You will prove Theorem 5-10 in Exercise 40.

**Problem 2 Using Theorem 5-10**

A town park is triangular. A landscape architect wants to place a bench at the corner with the largest angle. Which two streets form the corner with the largest angle?

MLK Blvd. + Valley Rd.



Got It? 2. Suppose the landscape architect wants to place a drinking fountain at the corner with the second largest angle. Which two streets form the corner with the second-largest angle?

MLK Blvd. + Hollingsworth Rd.



Take note

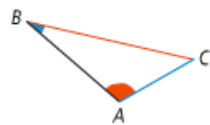
Theorem 5-11

Theorem

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

If ...

$$m\angle A > m\angle B$$

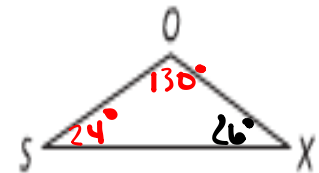


Then ...

$$BC > AC$$



Got It? 3. **Reasoning** In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Explain your reasoning.



$$m\angle X + 24 + 130 = 180$$

$$m\angle X + 154 = 180$$

$$m\angle X = 26^\circ$$

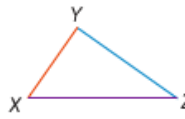
\overline{OX} is the shortest side because it is across from the smallest \angle

Take note

Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$XY + YZ > XZ \quad YZ + XZ > XY \quad XZ + XY > YZ$$



You will prove Theorem 5-12 in Exercise 45.



Got It? 4. Can a triangle have sides with the given lengths? Explain.

a. 2 m, 6 m, and 9 m

b. 4 yd, 6 yd, and 9 yd

$$2 + 6 > 9 \quad \times$$

not a Δ

$$4 + 6 > 9 \quad \checkmark$$

$$6 + 9 > 4 \quad \checkmark$$

$$4 + 9 > 6 \quad \checkmark$$

is a Δ



Got It? 5. A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?

4, 7, x

$$4 + 7 > x$$

$$11 > x$$

$$4 + x > 7$$

$$-4 \quad -4$$

$$x > 3$$

~~$$7 + x > 4$$~~

~~$$-7 \quad -7$$~~

~~$$x > -3$$~~

$$3 < x < 11$$

Name

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10-20 even

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Notes 5.7