5-3 Bisectors in Triangles

Content Standard
G.C.3 Construct the inscribed and circumscribed circles of a triangle . . .

Objective To identify properties of perpendicular bisectors and angle bisectors

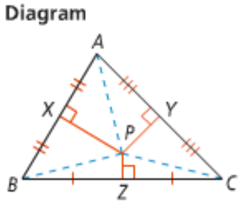
In the Solve It, the three lines you drew intersect at one point, the center of the circle. When three or more lines intersect at one point, they are **concurrent**. The point at which they intersect is the **point of concurrency**.

take note

Theorem 5-6 Concurrency of Perpendicular Bisectors Theorem

Theorem
The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

Diagram

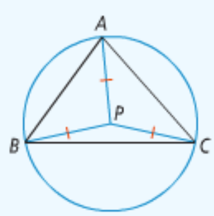


Symbols
Perpendicular bisectors $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are concurrent at P.

$PA = PB = PC$

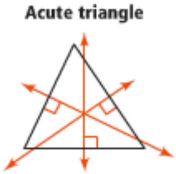
The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle.**

Since the circumcenter is equidistant from the vertices, you can use the circumcenter as the center of the circle that contains each vertex of the triangle. You say the circle is **circumscribed about** the triangle.

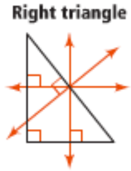


The circumcenter of a triangle can be inside, on, or outside a triangle.

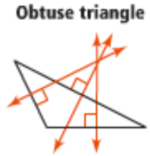
Acute triangle



Right triangle



Obtuse triangle



Draw the perpendicular bisectors of the sides of the triangle.

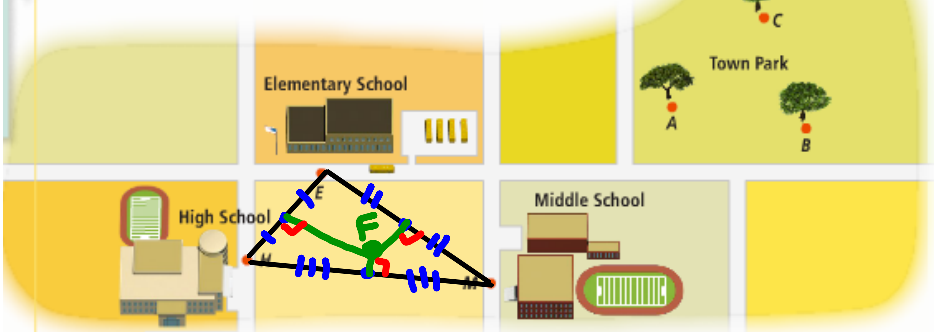
They are concurrent at a point - circumcenter

The distance from the circumcenter to each vertex of the triangle is equidistant.

The circle is circumscribed (outside) about the triangle.

Problem 2 Using a Circumcenter

A town planner wants to locate a new fire station equidistant from the elementary, middle, and high schools. Where should he locate the station?



The fire station should be placed at the circumcenter which is where the ⊥ bisectors of the Δ meet.

November 8, 2017

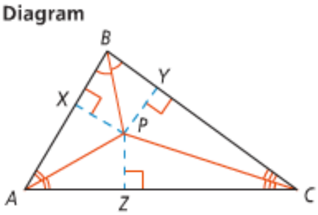
take note

Theorem 5-7 Concurrency of Angle Bisectors Theorem

Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Diagram



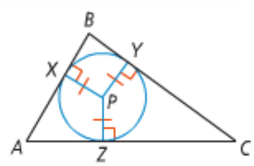
Symbols

Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at P.

$PX = PY = PZ$

You will prove Theorem 5-7 in Exercise 24.

The point of concurrency of the angle bisectors of a triangle is called the **incenter of the triangle.** For any triangle, the incenter is always inside the triangle. In the diagram, points X, Y, and Z are equidistant from P, the incenter of $\triangle ABC$. P is the center of the circle that is **inscribed in** the triangle.



Draw the angle bisectors of the triangle.

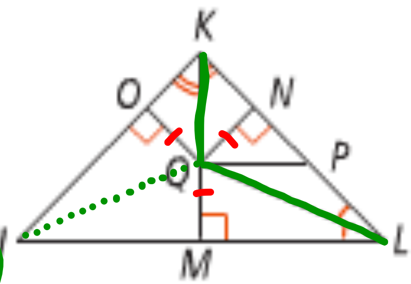
They are concurrent at a point - incenter

The distance from the incenter to each side of the triangle

(perpendicular) is equidistant.

The circle is inscribed (inside) in the triangle.

Got It? **3. a.** $QN = 5x + 36$ and $QM = 2x + 51$. What is QO?



Q is an incenter
(created by angle bisectors)

$QM = QN = QO$

$5x + 36 = 2x + 51$

$3x + 36 = 51$

$3x = 15$

$x = 5$

$QO = 2(5) + 51$

$QO = 61$

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Notes 5.4