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# Congruence in Overlapping Triangles

**Content Standard**  
**G.SRT.5** Use congruence . . . criteria to solve problems and prove relationships in geometric figures.

**Objectives** To identify congruent overlapping triangles  
 To prove two triangles congruent using other congruent triangles



**Problem 1 Identifying Common Parts**

What common angle do  $\triangle ACD$  and  $\triangle ECB$  share?

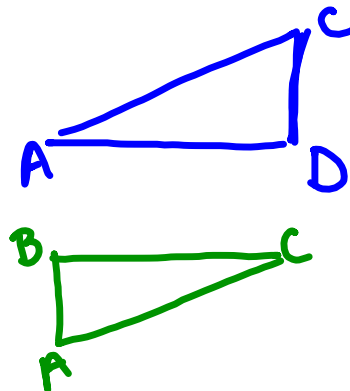
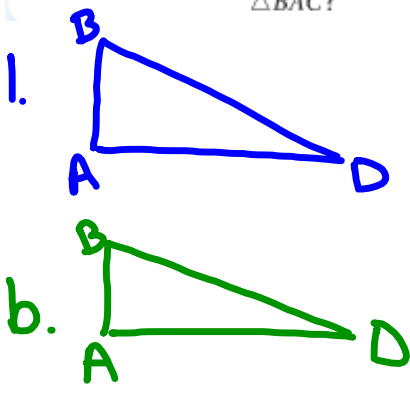
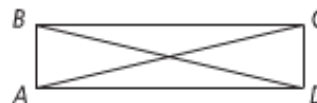


Separate and redraw  $\triangle ACD$  and  $\triangle ECB$ .



The common angle is  $\angle C$ .

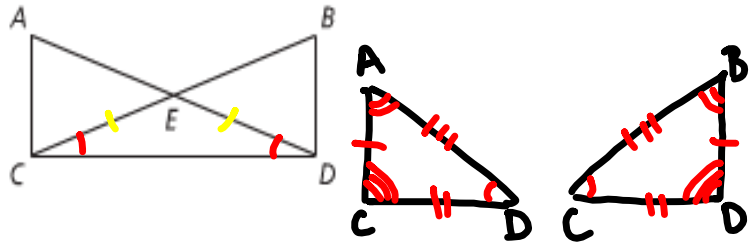
- Got It?** 1. a. What is the common side in  $\triangle ABD$  and  $\triangle DCA$ ?  
 b. What is the common side in  $\triangle ABD$  and  $\triangle BAC$ ?



$\overline{AD}$

$\overline{BA}$

Got It? 2. Given:  $\triangle ACD \cong \triangle BDC$   
 Prove:  $\overline{CE} \cong \overline{DE}$



Statements

Reasons

1.  $\triangle ACD \cong \triangle BDC$

1. Given

2.  $\angle ADC \cong \angle BCD$

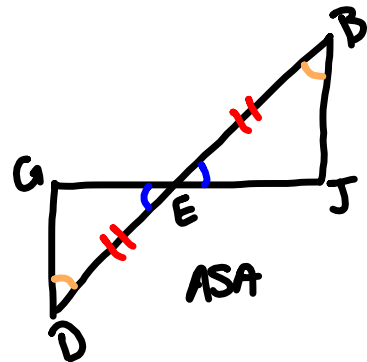
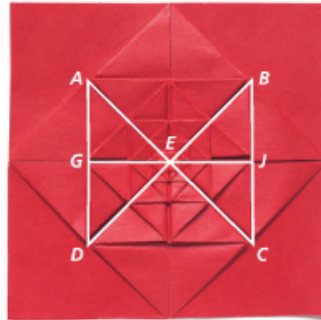
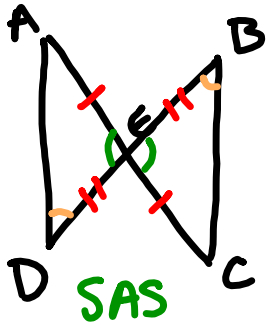
2. CPCTC

3.  $\overline{CE} \cong \overline{DE}$

3. Converse of the  
 Isosceles  $\triangle$  Thm

**Given:** In the origami design,  $E$  is the midpoint of  $\overline{AC}$  and  $\overline{DB}$ .

**Prove:**  $\triangle GED \cong \triangle JEB$



Statements

Reasons

1.  $E$  is the mdpt of  $\overline{AC}$   
 $\overline{DB}$

1. Given

2.  $\overline{AE} \cong \overline{CE}$   
 $\overline{DE} \cong \overline{BE}$

2. def. of midpoint

3.  $\angle AED \cong \angle CEB$

3. Vertical  $\angle$ s Thm

4.  $\triangle AED \cong \triangle CEB$

4. SAS

5.  $\angle D \cong \angle B$

5. CPCTC

6.  $\angle GED \cong \angle JEB$

6. Vertical  $\angle$ s Thm

7.  $\triangle GED \cong \triangle JEB$

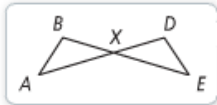
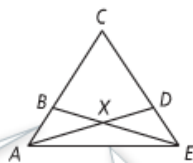
7. ASA



**Problem 4** Separating Overlapping Triangles

**Given:**  $\overline{CA} \cong \overline{CE}$ ,  $\overline{BA} \cong \overline{DE}$

**Prove:**  $\overline{BX} \cong \overline{DX}$



**Statements**

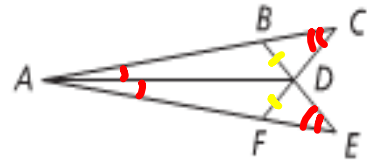
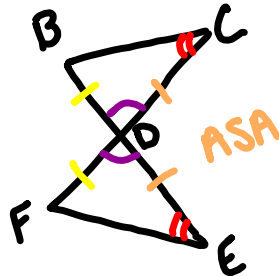
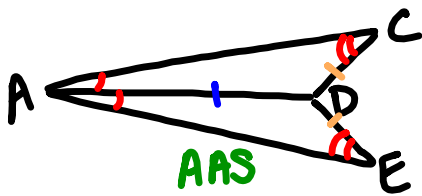
- 1)  $\overline{BA} \cong \overline{DE}$
- 2)  $\overline{CA} \cong \overline{CE}$
- 3)  $\angle CAE \cong \angle CEA$
- 4)  $\overline{AE} \cong \overline{AE}$
- 5)  $\triangle BAE \cong \triangle DEA$
- 6)  $\angle ABE \cong \angle EDA$
- 7)  $\angle BXA \cong \angle DXE$
- 8)  $\triangle BXA \cong \triangle DXE$
- 9)  $\overline{BX} \cong \overline{DX}$

**Reasons**

- 1) Given
- 2) Given
- 3) ~~Base angles of an isosceles triangle are congruent~~ **Isosceles  $\Delta$  Thm**
- 4) Reflexive Property of  $\cong$
- 5) SAS
- 6) Corresp. parts of  $\cong \Delta$  are  $\cong$ . **CPCTC**
- 7) ~~Vertical angles are congruent~~ **Vertical  $\angle$  Thm**
- 8) AAS
- 9) ~~Corresp. parts of  $\cong \Delta$  are  $\cong$~~  **CPCTC**

**Got It?** 4. Given:  $\angle CAD \cong \angle EAD, \angle C \cong \angle E$

Prove:  $\overline{BD} \cong \overline{FD}$



Statements

Reasons

1.  $\angle CAD \cong \angle EAD$

1. Given

$\angle C \cong \angle E$

2.  $\overline{AD} \cong \overline{AD}$

2. Reflexive Prop. of  $\cong$

3.  $\triangle ADC \cong \triangle ADE$

3. AAS

4.  $\overline{CD} \cong \overline{ED}$

4. CPCTC

5.  $\angle BDC \cong \angle FDE$

5. Vertical  $\angle$ s Thm

6.  $\triangle BDC \cong \triangle FDE$

6. ASA

7.  $\overline{BD} \cong \overline{FD}$

7. CPCTC

Name

4.7

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