2-5 Reasoning in Algebra and Geometry

© Content Standards
Prepares for G.CO.9 Prove theorems about lines and angles.
Prepares for G.CO.10 Prove theorems about triangles.
Prepares for G.CO.11 Prove theorems about parallelograms.

Objective To connect reasoning in algebra and geometry

Essential Understanding Algebraic properties of equality are used in geometry. They will help you solve problems and justify each step you take.

In geometry you accept postulates and properties as true. Some of the properties that you accept as true are the properties of equality from algebra.

take note

Key Concept Properties of Equality

Let a, b, and c be any real numbers.

|	
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then b can replace a in any expression.

© Problem 1 Justifying Steps When Solving an Equation

Algebra What is the value of x? Justify each step.

$\angle AOM$ and $\angle MOC$ are supplementary. △ that form a linear pair are supplementary.

$(2x + 30)^\circ$ x° at M, A, O, C

$m\angle AOM + m\angle MOC = 180$ Definition of supplementary △

$(2x + 30) + x = 180$ Substitution Property

$3x + 30 = 180$ ~~Distributive Property~~ *combine like terms/simplify*

$3x = 150$ Subtraction Property of Equality

$x = 50$ Division Property of Equality
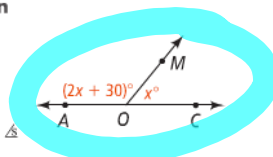
✓ **Got It?** 1. What is the value of x? Justify each step.
Given: $\overrightarrow{AB}$ bisects $\angle RAN$.

x° $(2x - 75)^\circ$ at B, R, A, N
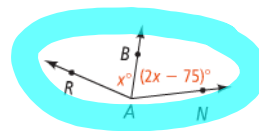
Statement	Reason
1. $\angle RAB = \angle BAN$	1. def. of bisect
2. $x = 2x - 75$ $-2x$ $-2x$	2. Substitution
3. $-x = -75$ $\div -1$ $\div -1$	3. Subtraction Prop. of =
4. $x = 75$	4. Division Prop. of =

take note

Key Concept   Properties of Congruence

Reflexive Property	$\overline{AB} \cong \overline{AB}$ $\angle A \cong \angle A$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.
	If $\angle B \cong \angle A$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Ⓒ **Problem 2**   **Using Properties of Equality and Congruence**

What is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

A $2x + 9 = 19$

$\qquad 2x = 10$ 　　　　　　　Subtraction Property of Equality

B $\angle O \cong \angle W$ and $\angle W \cong \angle L$

$\qquad \angle O \cong \angle L$ 　　　　　　Transitive Property of Congruence

C $m\angle E = m\angle T$

$\qquad m\angle T = m\angle E$ 　　　　　Symmetric Property of Equality

✅ **Got It? 2.** For parts (a)–(c), what is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

a. $\overline{AR} \cong \overline{TY}$ 　　　　　**b.** $3(x + 5) = 9$ 　　　　**c.** $\frac{1}{4}x = 7$

$\quad \overline{TY} \cong \overline{AR}$ 　　　　　　　$3x + 15 = 9$ 　　　　　　$x = 28$

d. Reasoning What property justifies the statement $m\angle R = m\angle R$?

a. Symmetric Prop. of $\cong$
b. Distributive Prop. of $=$
c. Multiplication Prop. of $=$
d. Reflexive Prop. of $=$

A **proof** is a convincing argument that uses deductive reasoning. A proof logically shows why a conjecture is true. A **two-column proof** lists each statement on the left. The justification, or the reason for each statement, is on the right. Each statement must follow logically from the steps before it.

Problem 3   Writing a Two-Column Proof

Write a two-column proof.

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle AEC = m\angle DEB$
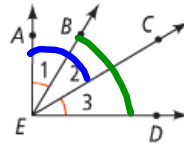
Know
$m\angle 1 = m\angle 3$

Need
To prove that $m\angle AEC = m\angle DEB$

Plan
Add $m\angle 2$ to both $m\angle 1$ and $m\angle 3$. The resulting angles will have equal measure.

Statements	Reasons
1) $m\angle 1 = m\angle 3$	1) Given
2) $m\angle 2 = m\angle 2$	2) Reflexive Property of Equality
3) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3) Addition Property of Equality
4) $m\angle 1 + m\angle 2 = m\angle AEC$ $m\angle 3 + m\angle 2 = m\angle DEB$	4) Angle Addition Postulate
5) $m\angle AEC = m\angle DEB$	5) Substitution Property

Got It? 3. **a.** Write a two-column proof.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$
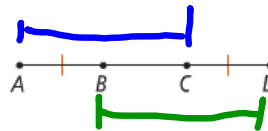
b. Reasoning In Problem 3, why is Statement 2 necessary in the proof?

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2. Reflexive Prop. of $\cong$
3. $\overline{AB} + \overline{BC} \cong \overline{BC} + \overline{CD}$	3. Addition Prop. of $\cong$
4. $\overline{AB} + \overline{BC} \cong \overline{AC}$ $\overline{BC} + \overline{CD} \cong \overline{BD}$	4. Segment Addition Post.
5. $\overline{AC} \cong \overline{BD}$	5. Substitution

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