

6-6

Trapezoids and Kites

© Content Standard

G.SRT.5 Use congruence . . . criteria to solve problems and prove relationships in geometric figures.

Objective To verify and use properties of trapezoids and kites

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called **bases**. The nonparallel sides are called **legs**. The two angles that share a base of a trapezoid are called **base angles**. A trapezoid has two pairs of base angles.



An **isosceles trapezoid** is a trapezoid with legs that are congruent. $ABCD$ at the right is an isosceles trapezoid. The angles of an isosceles trapezoid have some unique properties.



Take note

Theorem 6-19

Theorem

If a quadrilateral is an **isosceles trapezoid**, then each pair of base angles is congruent.

If . . .

$TRAP$ is an isosceles trapezoid with bases \overline{RA} and \overline{TP}

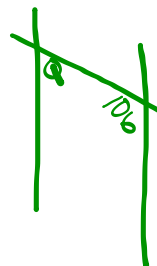
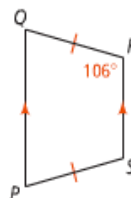
**Then . . .**

$\angle T \cong \angle P$, $\angle R \cong \angle A$



Got It? 1. a. In the diagram, $PQRS$ is an isosceles trapezoid and $m\angle R = 106$. What are $m\angle P$, $m\angle Q$, and $m\angle S$?

$$\angle S = 106^\circ$$



$$m\angle Q + 106 = 180$$

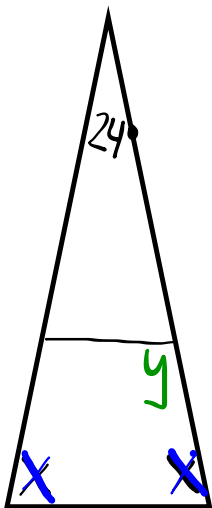
$$m\angle Q = 74^\circ$$

$$m\angle P = 74^\circ$$



Got It? 2. A fan like the one in Problem 2 has 15 angles meeting at the center. What are the measures of the base angles of the trapezoids in its second ring?

$$\angle 1 = \frac{360}{15} = 24^\circ$$



$$x + x + 24 = 180$$

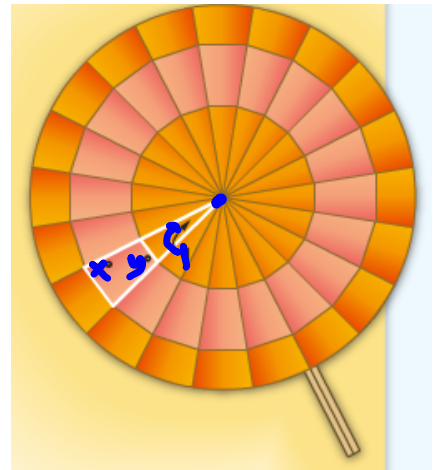
$$2x + 24 = 180$$

$$2x = 156$$

$$x = 78^\circ$$

$$y + 78 = 180$$

$$y = 102^\circ$$



Take note

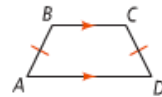
Theorem 6-20

Theorem

If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent.

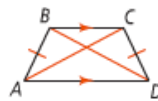
If ...

$ABCD$ is an isosceles trapezoid



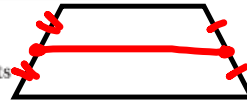
Then ...

$$\overline{AC} \cong \overline{BD}$$



You will prove Theorem 6-20 in Exercise 54.

In Lesson 5-1, you learned about midsegments of triangles. Trapezoids also have midsegments. The **midsegment of a trapezoid** is the segment that joins the midpoints of its legs. The midsegment has two unique properties.



Take note

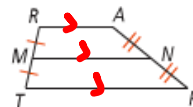
Theorem 6-21 Trapezoid Midsegment Theorem

Theorem

If a quadrilateral is a trapezoid, then
 (1) the midsegment is parallel to the bases, and
 (2) the length of the midsegment is half the sum of the lengths of the bases.

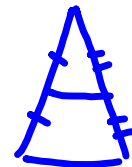
If ...

$TRAP$ is a trapezoid with midsegment \overline{MN}



Then ...

- (1) $\overline{MN} \parallel \overline{TP}$, $\overline{MN} \parallel \overline{RA}$, and
- (2) $MN = \frac{1}{2}(TP + RA)$



Got It? 3. a. Algebra \overline{MN} is the midsegment of trapezoid $PQRS$. What is x ? What is MN ?

$$\text{mdsg} = \frac{1}{2}(\text{base} + \text{base})$$

$$2x + 11 = \frac{1}{2}(8x - 12 + 10)$$

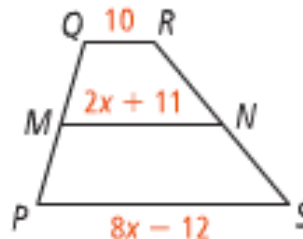
$$2x + 11 = \frac{1}{2}(8x - 2)$$

$$2x + 11 = 4x - 1$$

$$11 = 2x - 1$$

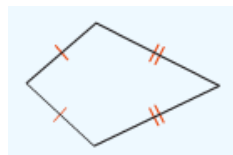
$$12 = 2x$$

$$6 = x$$



$$2(6) + 11 = 23 = MN$$

A **kite** is a quadrilateral with two pairs of consecutive sides congruent and no opposite sides congruent.



Take note

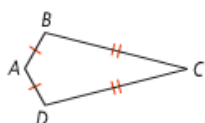
Theorem 6-22

Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

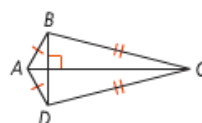
If ...

$ABCD$ is a kite



Then ...

$\overline{AC} \perp \overline{BD}$



Got It? 4. Quadrilateral $KLMN$ is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?

$$\angle 1 = 90^\circ$$

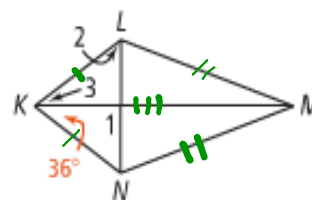
$$\angle 3 = 36^\circ$$



$$36 + 90 + m\angle 2 = 180$$

$$126 + m\angle 2 = 180$$

$$m\angle 2 = 54^\circ$$



SSS
CPCCTC

Name

b.6

pg. 394-396 # 8-24 even

28-36 even

39-44

47-52

57-62

December 6, 2017

